7.B.2
8.A.8

Nuclear Physics B14 (1969) 683-698. North-Holland Publ. Comp., Amsterdam

RESONANCE EXTINCTION AND EFFECTS OF DRIVING FORCES IN FINAL STATE INTERACTIONS

I. J. R. AITCHISON
Department of Theoretical Physics, Parks Road, Oxford, England

and

P. R. GRAVES-MORRIS,
Department of Applied Mathematics, University of Kent, Canterbury, Kent, England

Received 1 August 1969

Abstract: The absence of the rho resonance in the $\pi^+\pi^-$ subsystem of the $\pi^+\pi^-\pi^-$ final state produced by $\bar{p}n$ annihilation has prompted the investigation of resonance extinction in final state interactions. Resonance extinction is shown to be a general feature of a simple dispersion theoretic model. The qualities of the two-particle amplitude necessary for the corresponding decay amplitude not to have resonance extinction are discussed. An important factor throughout is the structure of the vertex function, i.e. the decay amplitude with no final state interaction. We re-examine, using explicit models of the two-particle amplitude and vertex functions with structure, the widespread conjecture due originally to Watson, that attractive forces enhance a decay amplitude.

1. INTRODUCTION

In weak decays, typified by a point production vertex in configuration space (the equivalent of a constant matrix element in momentum space), it has been suggested that resonances expected in final states may in some circumstances be extinguished. This means that the theory implies that two-particle resonances expected in a multiparticle final state would be absent. This has been shown analytically and numerically in an on mass shell theory [1] which describes the decay process 1 particle → 3 particles; but it is important to notice that the same cancellation would occur if only two of the particles in the final state interacted. This result, though, seems to depend on the particular model of final state interactions (f.s.i.) considered which is a K-matrix model. In particular, the literature does not answer the question whether the phenomenon can occur in ordinary two-particle f.s.i. using dispersion methods. This question is especially interesting in view of the long-known and unexplained absence of the ρ resonance in the reaction $\bar{p}n \rightarrow \pi^+\pi^-\pi^-$ (ref. [2]). While not providing a complete dynamical explanation, we will show that such cancellations are possible in the two-particle f.s.i. case, within a dispersion theoretic approach.

When such cancellations are taking place, we find that the details of the weak production vertex, or source, become important. A similar sensitivity to the details of the source has recently been reported by Amado and Noble [3]: they calculated the effect of strong attractive pairwise f.s.i. among three particles on the total rate for the decay of one particle into three, using separable potential theory. Among other interesting results, they find that strong enhancements or de-enhancements can only be produced in this case when the weak vertex is strongly localised in configuration space. This result pertains to the 3-particle f.s.i. case, but one immediately asks, especially in view of our results about cancellations, if a similar statement can be made in the simpler two-particle f.s.i. case, where the third particle does not interact. The role of the vertex function in f.s.i. is the second main topic of this paper.

Most of the remarks made in this paper apply, with slight modifications of the conditions, to multiparticle final states. The remarks also apply very directly to the three-point function, if the bare vertex in the absence of f.s.i. is expected to be non-zero. We also examine the current conjecture, due to some remarks by Watson [4], that attractive final state interactions enhance a decay amplitude and that repulsive f.s.i. reduce it. This certainly seems to be justified with a scattering length approximation and an effective range approximation model of the two-particle interaction.

2. EXTINCTION

To begin with, consider the decay of one particle into three. There are two sets of graphs which are thought to describe the f.s.i. problem. They are drawn schematically in fig. 1 for the case of two simultaneous f.s.i.. One either uses the set proportional to the ones in fig. 1b, or the entire set of fig. 1. Both appear in the literature, but we prefer the latter for reasons to be stated.

Numerous authors have found two features of f.s.i. to be most striking. The first is that f.s.i. can in some circumstances and in some theories be

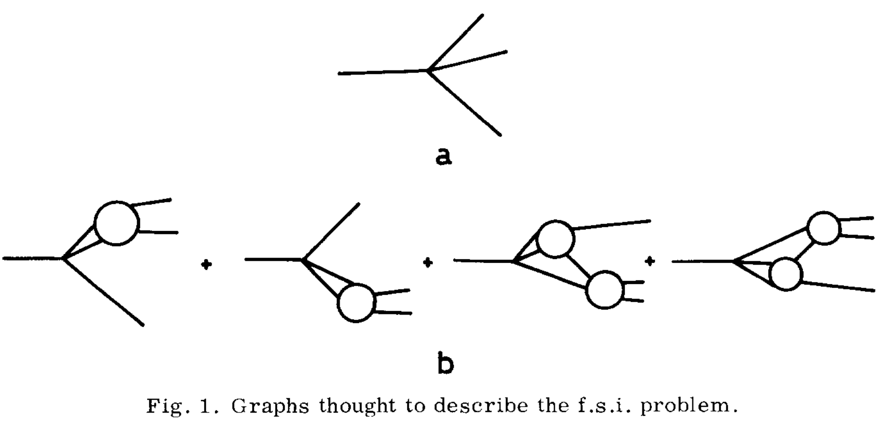

Fig. 1. Graphs thought to describe the f.s.i. problem.

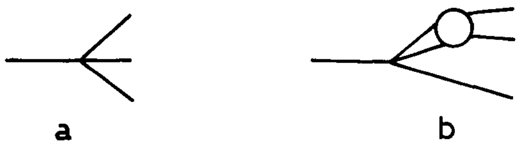

Fig. 2. Additional graphs needed to describe the f.s.i. problem.

gigantic effects of three orders of magnitude; the second is that rescattering singularities (due to processes like the third graph of fig. 2b) have not appeared conspicuously in elementary particles processes (although an interesting example is known in nuclear physics [5]). Everyone knows that two-particle resonances usually turn up in final states with roughly the expected masses and widths, so empirically one knows that multiple f.s.i. (i.e. rescattering effects) are not very significant, except possibly if the rescattering effects are so strong as to produce three-body resonances [6]. So the dominant graphs determining the important structure of the f.s.i. of particles 1 and 2 are shown in figs. 2a and 2b. In these figures, we are dropping the term present in fig. 1b involving 2-3 scattering, since we want to examine the questions raised in the introduction in the simplest case possible - that in which only two particles in fact interact strongly in the final state. Even in this simple and supposedly well understood case some interesting results appear.

With all these simplifications the decay amplitude $F(k^2)$ is given by

$$F(k^2) = b(k^2) + \frac{1}{\pi} \int_0^\infty \frac{g^*(q^2)F(q^2)}{q^2 - k^2 - i\epsilon}\, dq^2 , \tag{1}$$

where $g(q^2)$ is the two-particle s-wave amplitude, normalised so that $g(q^2) = \exp(i\delta)\sin(\delta)$ and $b(k^2)$ is the production vertex, the projection of fig. 2a. The quantity k is the c.m. momentum of one of the two equal-mass particles in the interacting two-particle subsystem.

A convenient resonant form with the correct analytic properties in the physical region is

$$g(k^2) = \frac{\Gamma k}{k_R^2 - k^2 - i\Gamma k} . \tag{2}$$

The solution of eq. (1) was shown by Omnès [7] to be

$$F(k^2) = b(k^2) + \frac{1}{\pi D(k^2)} \int_0^\infty \frac{D(q^2)g(q^2)b(q^2)}{q^2 - k^2 - i\epsilon}\, dq^2 , \tag{3}$$

when the integral converges.

The amplitude $D(k^2)$ is given by

$$D(k^2) = \exp\left(\frac{-1}{\pi} \int_0^\infty \frac{\delta(q^2)}{q^2 - k^2 - i\epsilon}\, dq^2\right) = \exp\Delta(k^2) , \tag{4}$$

where $D(k^2)$ needs only to be determined up to a constant multiplicative factor, and so subtractions in the dispersion integral for $\Delta(s)$ are permitted.

With one subtraction, Bronzan [8] showed that

$$D(k^2) = k_R^2 - k^2 - i\Gamma k \tag{5}$$

for the resonance form of eq. (2). If this is substituted directly into eq. (3), the integral does not converge for constant b, so proceed by subtracting eq. (1) at $k^2 = 0$.

$$\frac{F(k^2)}{k^2} = \frac{F(0)}{k^2} + \frac{b(k^2) - b(0)}{k^2} + \frac{1}{\pi}\int_0^\infty \frac{g^*(q^2)F(q^2)}{q^2(q^2 - k - i\epsilon)}\,dq^2 .$$

This is also an equation of Omnès type, whose solution, using eqs. (3) and (5) is

$$F(k^2) = F(0) + b(k^2) - b(0) + \frac{k^2}{\pi D(k^2)}\int_0^\infty \frac{\Gamma q(F(0) + b(q^2) - b(0))}{q^2(q^2 - k^2 - i\epsilon)}\,dq^2 .$$

If $b(k^2)$ is constant, this converges and

$$F(k^2) = F(0)\,\frac{k_R^2 - k^2}{k_R^2 - k^2 - i\Gamma k} . \tag{6}$$

This is the solution and it equals $F(0)$ at $k^2 = 0$ and $k^2 = \infty$. One can evaluate $F(0)$ by backsubstitution of eq. (6) in eq. (1) to find that

$$F(k^2) = b + \frac{F(0)}{\pi}\int_0^\infty \frac{i\Gamma q(k_R^2 - q^2)}{(k_R^2 - q^2)^2 + \Gamma^2 q^2}\,\frac{dq^2}{q^2 - k^2} ,$$

where the integral is convergent. One obtains the fundamental form (6) again with

$$F(k^2) = b\,\frac{k_R^2 - k^2}{k_R^2 - k^2 - i\Gamma k} , \tag{7}$$

showing that $F(0) = F(\infty) = b$.

An alternative way to reach eq. (7) is to note from eq. (6) that $F(\infty) = F(0)$, so that the integral in eq. (1), together with $g(k^2)$ as given by eq. (2), converges, proving that $F(\infty) = b$ for a constant vertex part.

Before considering the implications of eq. (7), let us first find the solution for the case when $b(k^2)$ is not constant. A convenient choice is

$$b(k^2) = \frac{b\beta^2}{\beta^2 + k^2} . \tag{8}$$

Now the integral in eq. (3) converges, and we might be tempted to think that the correct solution was, using eq. (3),

$$F(k^2) = \frac{b\beta^2}{\beta^2+k^2}\,\frac{k_R^2 - k^2 + \Gamma\beta}{k_R^2 - k^2 - i\Gamma k}\,. \tag{9}$$

However, this cannot be the correct solution, since eq. (8) reduces to $b(k^2) = b$ = constant in the limit $\beta \to \infty$, but eq. (9) does not reduce to eq. (7). The reason in that the procedure leading to eq. (9) is invalid in this limit; since above all we require continuity of solution, we must find the solution which does tend to eq. (7) as $\beta \to \infty$.

Let us use the subtracted form of the equations, given above. We then obtain

$$F(k^2) = \frac{[F(0)(\beta^2+k^2) - bk^2](k_R^2 - k^2) - \Gamma b\beta k^2}{(\beta^2+k^2)(k_R^2 - k^2 - i\Gamma k)}\,. \tag{10}$$

We see that $F(k^2) \to F(0) - b$ as $k^2 \to \infty$, so that the integral in eq. (1) converges and $F(\infty) = b(\infty) = 0$ for finite β. Hence $F(0) = b$, and eq. (10) reduces to

$$F(k^2) = \frac{b\beta^2}{\beta^2+k^2}\,\frac{k_R^2 - k^2 - k^2\Gamma\beta^{-1}}{k_R^2 - k^2 - i\Gamma k}\,, \tag{11}$$

which indeed reduces to eq. (7) as $\beta \to \infty$. From eq. (11), it follows that $F(0) = b$, while for finite β, $F(\infty) = 0$; on the other hand, for $b(k^2)$ = constant, $F(0) = b = F(\infty)$. It is now clear why eq. (9) was incorrect in the limit $\beta \to \infty$. As $\beta \to \infty$ from finite values, the equation with inhomogeneous term $b(k^2) = b\beta^2/(\beta^2+k^2)$ tends to the equation with inhomogeneous term $b(k^2) = b$; but the latter has the solution (7) with $F(0) = F(\infty) = b$. Hence we must ensure that for the case β finite we also have $F(0) = b$. The solution of eq. (9) does not satisfy this condition, so we have to add to it a constant multiple of $D^{-1}(k^2)$, the solution of the homogeneous equation. If the arbitrary constant is determined by $F(0) = b$, we recover precisely eq. (11), which is then, by Omnès work, the unique solution of the problem which $\to 0$ at ∞, for finite β, and has the value b at $k^2 = 0$.

Let us now examine the physical consequences of eq. (11). The most striking feature is the existence of a zero of the numerator at $k^2 = k_R^2(1+\Gamma/\beta)^{-1}$. Now β^{-1} measures the spatial extension of the source, β its extension in momentum space. If $\beta \gg \Gamma$, the numerator of eq. (11) vanishes close to the resonance energy; the resonance is extinguished in this model for a very localised source. On the other hand, if $\beta \ll \Gamma$ which corresponds to a diffuse source, the resonance will survive, though it will be distorted and damped. Some typical cases are shown in fig. 3a for $\Gamma = 1$ and in fig. 3b for $\Gamma = 0.5$. These show $|F(k^2)|^2$ plotted versus k^2, and $|g(k^2)|^2$ is

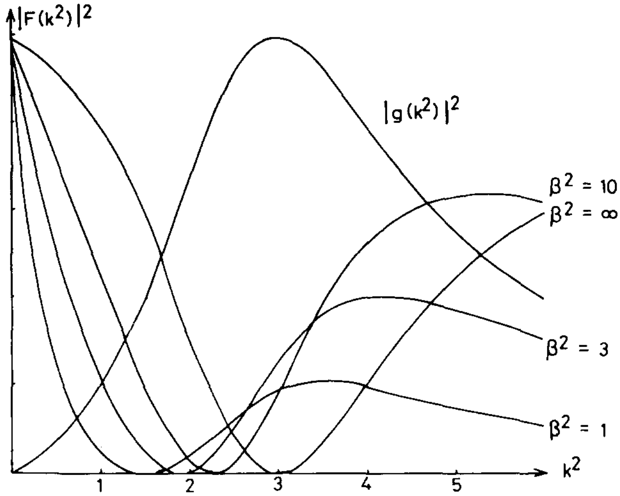

Figure 3a

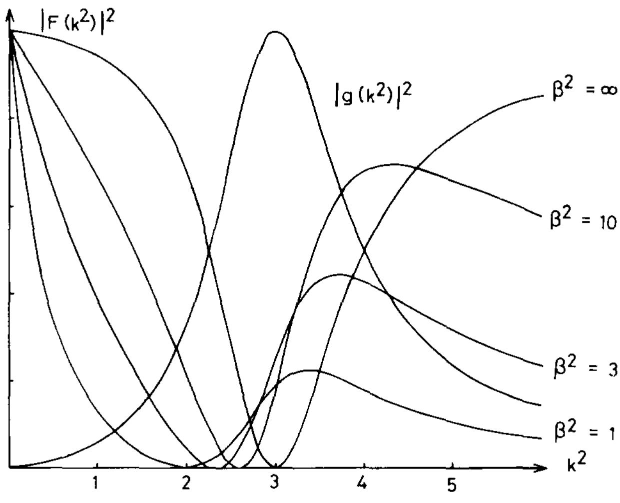

Figure 3b

Fig. 3. (a) Decay amplitudes with $\beta^2 = \infty$, 10, 3 and 1, $\Gamma = 1$; (b) decay amplitudes with $\beta^2 = \infty$, 10, 3 and 1, $\Gamma = 0.8$.

shown for comparison. The resonance energy is fixed at $k_R^2 = 3$, and β^2 is given values 1, 3, 10 and ∞.

It is clear that a spatially localized source leads, in this model, to distortion of a resonance. This is rather similar to the conclusion of Amado and Noble in the three-body case [3]. If β^2 is reduced, so that the form factor is stronger, the resonance is damped. We investigate, in sect. 4, to what extent these are model-dependent effects.

3. AN ALTERNATIVE MODEL OF TWO-PARTICLE F.S.I. THE HOMOGENEOUS SOLUTION

Before continuing further with the question of resonance extinction, we consider the alternative model mentioned earlier, in which the graphs of fig. 1b alone are considered, so that figs. 2a and 2b are replaced by fig. 2b alone. This model requires the solution of the homogeneous equation associated with eq. (1),

$$F(k^2) = \frac{1}{\pi} \int_0^\infty \frac{g^*(q^2)F(q^2)\mathrm{d}q^2}{q^2 - k^2 - i\epsilon} . \tag{12}$$

For $g(k^2)$ given by eq. (2), Omnès showed that [7]

$$F(k^2) = \frac{\Gamma}{k_\mathrm{R}^2 - k^2 - i\Gamma k} \times \text{regular function of } k^2 .$$

In this formulation, any resonance extinction has to be put in by hand, taking the regular function to have a zero at k_R^2, say. Dispersion theory need have nothing to say about the zeros of the amplitude, and in this model such a zero is certainly quite unrelated to the form of the decay vertex.

It is expected that a realistic two-particle amplitude $g(k^2)$ will tend to zero as $k^2 \to \infty$. If the vertex $b(k^2) = b$ = constant, one expects the solution of eq. (1) to tend to b as $k^2 \to \infty$, and the solution of the homogeneous eq. (12) to tend to zero as $k^2 \to \infty$ (ref. [9]).

Consider the case when the two-particle amplitude is generated by a potential which represents the exchange of some arbitrary particles. The diagram of fig. 2a represents the bare vertex, the decay Hamiltonian in Feynman theory. The diagram of fig. 2b is the sum of all Feynman graphs in which the two final state particles exchange particles. These graphs are the only ones with the two-particle normal threshold and which have an imaginary part associated with them. So the dispersion integral in eq. (12) gives the graphs contributing to fig. 2b. The problem is whether we should use eq. (1) or eq. (12) to describe decay amplitudes. If either equation is subtracted, one gets the same vertex function when the vertex function $b(s)$ is constant. This procedure simply evades the difficulty.

It seems to us that the first model (figs. 2a and 2b) based on the solution (3) is to be preferred because (a) the solution is manifestly proportional to the decay vertex if it is constant (b) when the decay vertex has structure, the amplitude does not have an additive part independent of this structure and (c) when the f.s.i. vanish, and the two-particle phase shift is zero, the decay amplitude is precisely the decay vertex.

4. SOLUTIONS WITH VARIOUS RESONANT TWO-PARTICLE AMPLITUDES

We return now to the question of extinction, using figs. 2a and 2b as a model. The resonant amplitude of eq. (2) has correct analytic properties on the right-hand cut, but has no left-hand cut at all. One might wonder if the cancellation was somehow due to this fact. In this section, it will be shown that the decay amplitude $F(k^2)$ does depend strongly on the precise form of the two-particle amplitude: resonance forms with resonance mass and widths equal simply do not produce similar forms for $F(k^2)$. A K-matrix model will exhibit the extinction phenomenon, and will show how the extinction zero moves when the left-hand cut is represented by a pole. On the other hand, one particular but fairly realistic N/D model of the two-particle amplitude gives little or no possibility of an extinction at all. The only possible conclusion of such an analysis is that until the forces responsible for actual physical two-particle resonances are understood, one can make no categorical statements about the necessary presence or absence of zeros in decay amplitudes. One can easily construct a unitary model of a two-particle amplitude by choosing a meromorphic K-matrix and checking that the resultant amplitude has no physical sheet poles except on the negative real axis. Use a K-matrix

$$K(k^2) = \frac{\tan\delta}{k} = \frac{a}{k_{\mathrm{R}}^2 - k^2} + \frac{c}{k^2 + b} ,$$

with $a, b > 0$ and a and c small. Then the resulting T-matrix is

$$T(k^2) = \frac{e^{i\delta}\sin\delta}{k} = \frac{ab + ck_{\mathrm{R}}^2 + k^2(a-c)}{(k_{\mathrm{R}}^2 - k^2)(k^2 + b) - ik(ab + ck_{\mathrm{R}}^2 + ak^2 - ck^2)} ,$$

which has two complex conjugate poles on the unphysical k^2 sheet near $k^2 = k_{\mathrm{R}}^2$ representing resonances. It has two poles near $k^2 = -b$ on both sheets, and the one on the physical sheet represents a weak left-hand cut. If c is positive, it represents attractive forces, and if c is negative, repulsive forces. If the pole in this latter case was itself generated by forces, then the pole might be interpreted as a bound state, but in this context the pole itself is representing the forces and the fact that this pole looks like a bound state should not be taken literally.

Suppose that the physical sheet pole of $T(k^2)$ occurs at $k^2 = k_{\mathrm{B}}^2$. Then the Omnès D function is found to be

$$D(k^2) = [(k_{\mathrm{R}}^2 - k^2)(k^2 + b) - ik(ak^2 + ab + ck_{\mathrm{R}}^2 - ck^2)](k^2 - k_{\mathrm{B}}^2)^{-1}$$

and the solution for $F(k^2)$ with a constant vertex part $b(k^2) = b_{\mathrm{V}}$ is

$$F(k^2) = b_{\mathrm{V}} \frac{(k_{\mathrm{R}}^2 - k^2)(k^2 + b) + k^2 |k_{\mathrm{B}}|^{-1}(ak_{\mathrm{B}}^2 + ab + ck_{\mathrm{R}}^2 - ck_{\mathrm{B}}^2)}{(k_{\mathrm{R}}^2 - k^2)(k^2 + b) - ik(ak^2 + ab + ck_{\mathrm{R}}^2 - ck_{\mathrm{B}}^2)} .$$

The decay amplitude $F(k^2)$ has the phase of the two-particle amplitude and the extinction zero is seen to be shifted by a small amount to higher or lower energies according as to whether $(ak_{\mathrm{B}}^2 + ab + ck_{\mathrm{R}}^2 - k_{\mathrm{B}}^2 c)$ is positive or negative.

The decay probability $|F(k^2)|^2$ is shown plotted against k^2 in fig. 4, and the distortion caused by the zero is very clear; $|T(k^2)|^2$ is also shown for comparison. The resonance position was fixed at $k_R^2 = 3$, and representative parameters $a = 0.3$, $b = 3$ were chosen. Fig. 4a shows the case of $c = 0.3$ and fig. 4b the case of $c = -0.3$.

Although this K-matrix model describes an amplitude in which a left-hand cut as well as resonance poles are present, the resonance poles are not generated by the forces represented by the left-hand cut: they are poles of the CDD non-dynamical type.

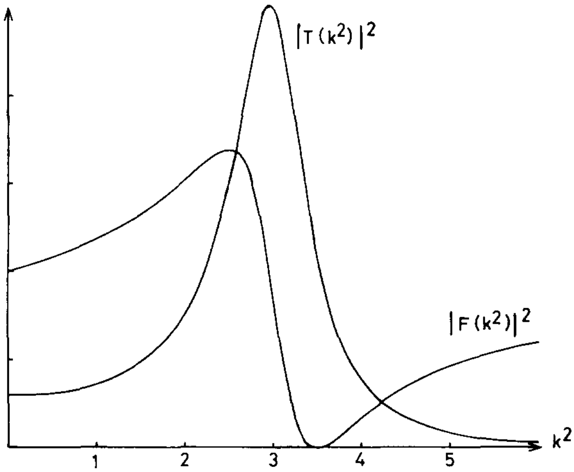

Figure 4a

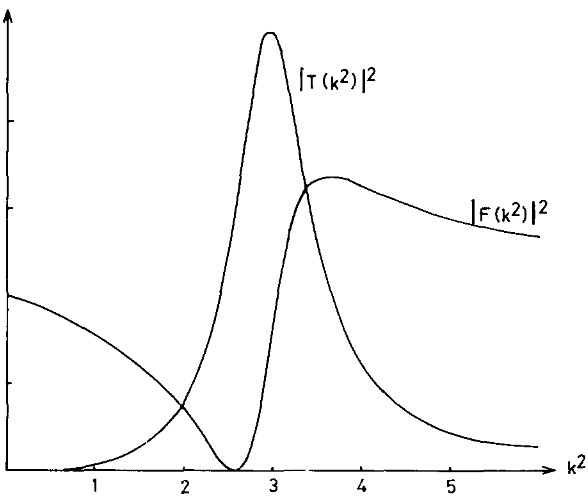

Figure 4b

Fig. 4. (a) Decay amplitude with K-matrix model and $a = 0.3$, $b = 3.0$, $c = 0.3$ and $k_R^2 = 3.0$, (b) decay amplitude with K-matrix model and $a = 0.3$, $b = 3.0$, $c = -0.3$ and $k_R^2 = 3$.

In contrast, consider the P-wave N/D two-particle amplitude, in which a pole approximation for N represents driving forces. With sufficiently strong forces, this leads to an amplitude with a moving zero of D which can produce a resonance.

Take

$$N = \lambda a^{-1}(k^2 + a^2)^{-1} ,$$

$$\operatorname{Im} D = -k^3 N ,$$

$$D(0) = 1 ,$$

then the P-wave amplitude is

$$T(k^2) = \frac{e^{i\delta} \sin\delta}{k} = \frac{Nk^2}{D} = \frac{\lambda k^2 a^{-1}}{(a + ik)(a - ik - \lambda k^2 a^{-1})} ,$$

which has moving poles at

$$k_{\pm}(\lambda) = -a(i \pm \sqrt{(4\lambda - 1)})/2\lambda . \tag{13}$$

These poles are plotted in the complex k-plane in fig. 5. For $\lambda > 0.25$, the poles move round the circle and may be interpreted as resonances for $\lambda > 0.5$. There is the slightly curious feature of this 'one-pole' model that complex conjugate poles of T occur with $\operatorname{Re}(q^2) < 0$, when λ is in the range $0.25 < \lambda < 0.5$.

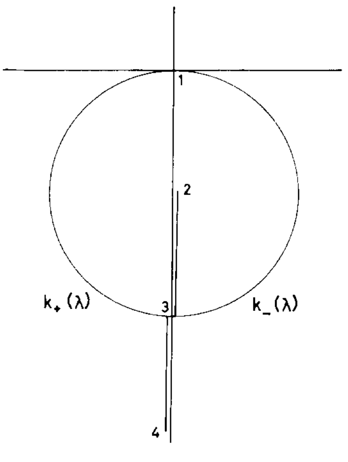

Fig. 5. The complex k plane: locus of zeros of D. 1. $\lambda = \infty$, $k_{\pm} = 0$, 2. $\lambda = 0$, $k_{-} = -ia$, 3. $\lambda = 0.25$, $k_{\pm} = -2ia$, 4. $\lambda = 0$, $k_{+} = -i\infty$.

The Omnès D function is

$$D(k^2) = (a - ik - \lambda k^2 a^{-1})(a + ik)^{-1} ,$$

and if we take a constant vertex b, the decay amplitude (ignoring the angular dependence for the P-wave in the final state) is

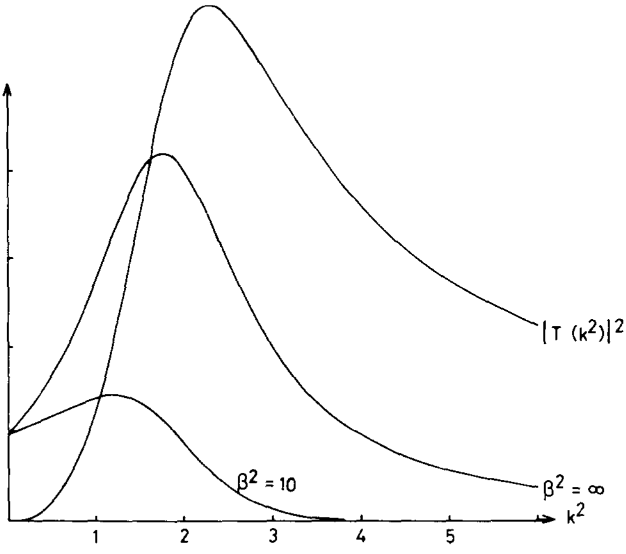

Figure 6a

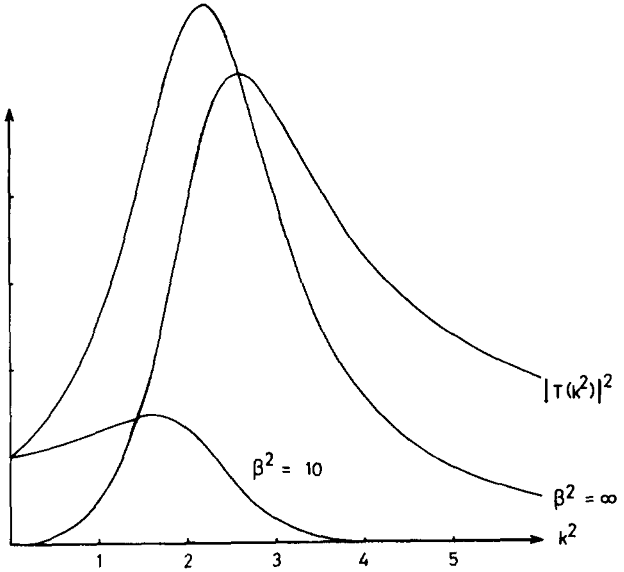

Figure 6b

Fig. 6. (a) Decay amplitudes in N/D model with $\lambda = 3.0$, $a = 2.45$; (b) Decay amplitudes in N/D model with $\lambda = 5.0$, $a = 3.46$, $\beta^2 = 10, \infty$.

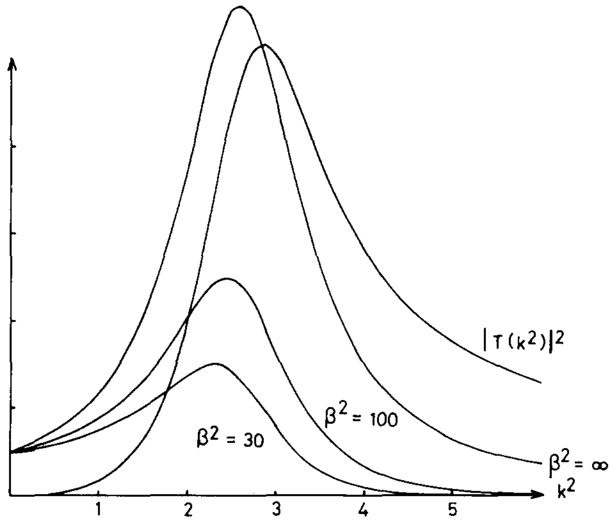

Fig. 6. (c) Decay amplitudes in N/D model with $\lambda = 10.0$, $a = 5.2$, $\beta^2 = \infty$, 100, 30.

$$F(k^2) = \frac{b(a - ik)}{a - ik - \lambda k^2 a^{-1}} . \tag{14}$$

There is no extinction in this case. For the case when the vertex has structure, we follow the procedure described in sect. 2, namely we choose that solution which reproduces eq. (14) as $\beta \to \infty$. This can be done by adding to solution (13) a constant multiple of D^{-1} so that the resultant expression tends to the same limit as $k \to 0$ as eq. (14) does, which is $F(0) = b$. This solution is

$$F(k^2) = \frac{b\beta^2}{\beta^2 + k^2} \frac{a - ik}{a - ik - \lambda k^2 a^{-1}} \left(1 - \frac{\lambda k^2}{a(a + \beta)}\right) ,$$

which indeed reduces to eq. (14) as $\beta \to \infty$. Extinction occurs when $\beta^2 = a^2/(\lambda - 1)^2$: the zero is at $k^2 = a(a + \beta)\lambda^{-1}$ and the phase shift increases through $\frac{1}{2}\pi$ at $k^2 = a^2/(\lambda - 1)$.

The results of a calculation are shown in fig. 6. We used $\lambda = 3$ in fig. 6a, $\lambda = 5$ in fig. 6b and $\lambda = 10$ in fig. 6c. The values of a are chosen so that resonance occurs at $k^2 = a^2/(\lambda - 1) = 3$. The decay probability $|F(k^2)|^2$ is peaked at energies a little less than the resonance energy when the vertex is constant ($\beta = \infty$); $|T(k^2)|^2$ is plotted for comparison. We note the strong damping caused by the form factor when $\beta^2 = 10$. It would seem, tentatively, that 'dynamically generated' resonances can be strongly damped by interference, while the necessary cancellations for complete extinction might easily occur for the non-dynamical or CDD type of resonance.

5. SOLUTIONS WITH NON-RESONANT TWO-PARTICLE AMPLITUDES SOURCE EFFECTS

Motivated by the foregoing results about possible extinction of two-body resonances by f.s.i. and the significance of the production vertex, it seems worthwhile to re-examine the general question of to what extent, if at all, the traditional statement that attractive two-body forces enhance decay amplitudes while repulsive ones suppress them is affected by introducing structure into the production vertex. It is interesting to recall that the above statement was first made by Watson [4], but with the proviso that the spatial extension of the secondary (final state) interactions be large compared with that of the primary (production) interaction. The notion that primary and secondary interactions are sequential is, of course, implicit. The dispersion technique is well suited to dealing with this question.

The two most popular parametrisations of low-energy amplitudes are used, namely the scattering length approximation (s.l.a.) and the effective range form. In the s.l.a., the s-wave two-particle amplitude is

$$T(k^2) = \frac{e^{i\delta}\sin\delta}{k} = \frac{a}{1-ika} ,$$

where $\tan\delta = ka$.

The s.l.a. is the amplitude deduced by taking $N = a$ = constant in the N/D method, and using one subtraction at threshold for D. So the case $a > 0$ implies attractive forces, a positive phase shift and a virtual bound state is present. If $a < 0$, one can say that there are repulsive forces and a negative phase shift. But how do we interpret the physical sheet pole? For the purposes of application, we can say *either* that the amplitude has a bound state *or* that the pole represents a left-hand cut. The s.l.a., being a crude one-parameter model, serves for both these cases. If a bound state exists as a physical particle, its pole should be present on the physical sheet of a decay amplitude. If the pole of the s.l.a. merely represents forces, then the f.s.i. model should not have such a pole on the physical sheet. A dispersion integral of the Omnès type gives a decay amplitude with a right-hand cut only: this is correct if the pole represents forces. If the pole represents a genuine bound state, we obtain the correct D function for this application by analytic continuation from positive to negative a. The case where the s.l.a. is a bound state model is denoted by BSM, and where the s.l.a. is a repulsive force model it is denoted by RFM.

The Omnès D functions are

$$D(k^2) = (1+ka)^{-1} \qquad a < 0 \text{ RFM}$$

$$D(k^2) = 1 - ika \qquad \left\{\begin{matrix} a \geq 0 \\ a < 0 \end{matrix}\right. \text{ BSM} .$$

The decay amplitudes are calculated with a vertex function

$$b(k^2) = b\beta^2/(\beta^2 + k^2) ,$$

and are

$$F(k^2) = \frac{b\beta^2}{\beta^2 + k^2} \quad \frac{1 + ika}{1 - \beta a} \qquad a < 0\,, \text{ RFM}$$

$$F(k^2) = \frac{b\beta^2}{\beta^2 + k^2} \quad \frac{1 + \beta a}{1 - ika} \qquad \left\{\begin{matrix} a > 0 \\ a < 0 \end{matrix}\right., \text{ BSM}\ .$$

So one sees that (a) Watson's theorem is obeyed, (b) attractive forces enhance the decay amplitude and repulsive forces reduce it, (c) the BSM has the physical sheet pole in the decay amplitude. We proceed with a more sophisticated model which removes the ambiguity in interpretation of the pole

The effective range approximation is a two-parameter amplitude and in the S-wave N/D formulation has a well-known analytic form [10]. The two-particle amplitude is

$$T(k^2) = \frac{e^{i\delta}\sin\delta}{k} = (k - ia)^{-1}\left(\frac{k + ia}{\lambda} + \frac{k - ia}{2a}\right)^{-1}\,,$$

and

$$k\cot\delta = a^2\left(\frac{1}{\lambda} - \frac{1}{2a}\right) + k^2\left(\frac{1}{\lambda} + \frac{1}{2a}\right)\,.$$

The amplitude has two poles representing left-hand cuts, at

$$k = ia \quad \text{and} \quad k = ia(\lambda - 2a)(\lambda + 2a)^{-1}\,.$$

The second pole is on the second k^2 sheet when $-2a < \lambda < 2a$, and we consider attractive forces with $\lambda > 0$ and also repulsive forces with $\lambda < 0$.

The Omnès D-function in these cases is

$$D(k^2) = \left(\frac{k + ia}{\lambda} + \frac{k - ia}{2a}\right)(k + ia)^{-1}\,.$$

The decay amplitude with a constant vertex is

$$F(k^2) = b\,\frac{(k + ia)(2a + \lambda)}{(k + ia)2a + \lambda(k - ia)}\,,$$

and with a momentum-dependent vertex

$$b(k^2) = \frac{b\beta^2}{k^2 + \beta^2}\,,$$

it is

$$F(k^2) = \frac{b\beta^2}{\beta^2 + k^2}\,\frac{k + ia}{(k + ia)2a + \lambda(k - ia)}\left(2a - \lambda + \frac{2\lambda\beta}{a + \beta}\right)\,.$$

One sees again that repulsive forces reduce the decay amplitude and attractive forces enhance it.

We note that for all these cases, we have the simple general result [11]

$$F(k^2) = b(k^2)\,\frac{D(-\beta^2)}{D(k^2)}\,.$$

6. CONCLUSION

The purpose of this paper was to consider certain simple models which have been thought to describe f.s.i. and to observe that good simple models of resonant two-particle amplitudes are not necessarily satisfactory for use in decay theory. This is upsetting because it was conjectured that a phenomenological parametrisation of two-particle amplitudes in the physical region might suffice to determine decay amplitudes. In particular, if model-dependent effects are important, then one must be suspicious of separable potential models which have bad analytic properties on the left-hand cut.

We find that with non-resonant two-particle amplitudes, the decay amplitude of one particle (to a multiparticle state with two strongly interacting particles in it) is enhanced by attractive forces and reduced by repulsive ones, whether the source has extension or not. In the case of resonant two-particle amplitudes, we saw that there is the possibility of resonance extinction, or of gross distortion of resonances when the zero is shifted a little from resonance, caused by destructive interference between figs. 2a and 2b. This may be a model-dependent effect, but it is a plausible model, and there is a significant piece of experimental evidence for the extinction of the rho resonance in the 3π mode of $\bar{p}n$ annihilation at rest.

All the remarks made in this paper, except sect. 3, refer to the graphs of fig. 2, which ignore graphs of the interactions of particle pairs 1, 3 and 2, 3. More generally, the remarks apply to a multiparticle final state with one pair of particles having a significant interaction. It may be a good approximation to ignore these other interactions for some pairs of physical particles. In the on-shell model [1], equal and simultaneous resonant interactions in all channels do not prevent the extinction of the resonance. In any case enhancements in the 1, 3 and 2, 3 subsystems appear averaged (over an angle variable) when events are plotted against the 1, 2 subenergy variable, so that their effect on the 1, 2 channel is likely to be smoothed. The projection ensures that reflected singularities are no worse than logarithmic.

REFERENCES

[1] P.R.Graves-Morris, Nuovo Cimento 50 (1967) 681.
[2] P.Anninos et al., Phys. Rev. Letters 20 (1968) 402.
[3] R.D.Amado and J.V.Noble, Phys. Rev. Letters 21 (1968) 1846.
[4] K.M.Watson, Phys. Rev. 88 (1952) 1163.
[5] J.Lang, R.Müller, W.Wölfli, R.Bösch and P.Marmier, Phys. Letters 15 (1965) 248.
[6] J.L.Basdevant and R.E.Kreps, Phys. Rev. 141 (1966) 1398; 141 (1966) 1404; 141 (1966) 1409.
[7] R.L.Omnès, Nuovo Cimento 8 (1958) 316.
[8] J.B.Bronzan, Phys. Rev. 134B (1964) 687.
[9] M.L.Goldberger, Dispersion relations and elementary particles, ed. R.Omnès and B.DeWit (Wiley, 1960).
[10] S.Frautschi, Regge poles and S-matrix theory (Benjamin, 1963).
[11] J.L.Basdevant, Lectures at Herceg Novi (1968), preprinted by Centre for Nuclear Research, Strasbourg.

NOTE ADDED IN PROOF

(i) After completion of this paper, we received a preprint entitled 'Rescattering, Omnès equation, and zeros in a resonance peak' by L. Resnick (Physics Department, Carleton University, Ottawa, Canada; May 27th 1969). This paper obtains a result similar to ours; namely, a zero at or near the elastic resonance maximum may occur in the f.s.i. problem when the elastic phase shift rises smoothly from 0 to π (tending to π at ∞). This is the behaviour of the phase shift given by our eq. (2). We are grateful to Dr. Resnick for correspondence about the problem.

(ii) We wish to add some further comments about CDD poles. Were Basdevant's [11] result, $F(k^2) = b(k^2)D(-\beta^2)/D(k^2)$, true in general, there could never be extinction. The reason it is not true in our case is that the amplitude given in eq. (2) corresponds to one CDD pole present. This may be seen from the well-known fact that $\delta(0)-\delta(\infty)$ = (number of bound states − number of CDD poles) $\times\ \pi$. In this situation the D dynamically generated by N, call it D_N, will *not* be the same as that which must be used in the f.s.i. problem, call it D_0, which is calculated from the phase shift δ via eq. (4). We have

$$D_N = 1 - \int \frac{N\rho\, \mathrm{d}q^2/\pi}{q^2 - k^2 - i\epsilon},$$

which, with a CDD pole present, is modified to

$$D_N^{\mathrm{CDD}} = 1 - \int \frac{N\rho\, \mathrm{d}q^2/\pi}{q^2 - k^2 - i\epsilon} + \frac{\gamma}{k^2 + k_c^2},$$

where γ, k_c^2 are the CDD parameters. Now it is known that $D_0 \propto$ $\propto (k^2 + k_c^2)D_N^{\mathrm{CDD}}$ (see E. J. Squires and P. B. Collins, Regge Poles in Particle Physics, Springer-Verlag, 1968, Ch. 6). So the f.s.i. problem, eq. (1), has the solution (as in eq. (3))

$$F = b + \frac{1}{\pi D_0}\int \frac{bgD_0\, \mathrm{d}q^2}{q^2 - k^2 - i\epsilon} = b\, \mathrm{e}^{i\delta}\left\{\cos\delta + \frac{1}{\pi |D_0|}P\int \frac{N\rho(q^2 + k_c^2)}{q^2 - k^2}\, \mathrm{d}q^2\right\},$$

where P stands for the principal value. The integral involving N is no longer simply related to D_0, which is the basis of Basdevant's result. Since $\cos\delta = 0$ when $\delta = \frac{1}{2}\pi$, it appears that it depends on the choice of the CDD parameters whether the extinction zero is at or near the resonance position, or not; in our example of sect. 2, and in Resnick's case, resonance extinction occurs. We are grateful to Professors Blankenbecler and Amado for emphasizing to us the importance of CDD poles in this problem.